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New superembeddings for type-II superstrings

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Abstract: Possible ways of generalization of the superembedding approach for the supersurfaces with the number of Grassmann directions being less than the half of that for the target superspace are considered on example of type-II superstrings. Focus is on $n = (1, 1)$ superworldsheet embedded into $D = 10$ type-II superspace that is of the interest for establishing a relation with the NSR string.

Keywords: Superstrings and Heterotic Strings, Supersymmetry Breaking, Supergravity Models.

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Contents

1. Introduction

The interpretation of superbranes as solitons of supergravity/super-Yang-Mills theories for the first time suggested by J. Hughes et.al. [1] (for reviews see [2]) allows to connect partial breaking of target space supersymmetries, caused by these solitonic configurations, with $\kappa-$symmetry inhereit to the brane worldvolume actions. In particular, for conventional superbranes that leave unbroken half of the target space supersymmetries $\kappa-$symmetry has the same number of independent parameters. In the proper gauge the action of $\kappa-$symmetry transformations for one half of the target space Grassmann coordinates η coincides with that of supersymmetry transformations. It is these coordinates that are associated with the unbroken part of target space supersymmetries and they can be identified with Grassmann coordinates on the superworldvolume. The other components of the target space Grassmann coordinates θ' constitute Goldstone fermions associated with the broken supersymmetries.

The worldvolume nature of $\kappa-$symmetry was revealed later by D.V. Volkov and collaborators [3, 4, 5]. (It should be noted, that the early attempts to combine the worldsheet and target space supersymmetries for particles and strings were made in the middle of 80's [6]. However, suggested method was too general to describe conventional models. For recent developments see [7].) This inspired the investigation of worldvolume superfield formulations of superbranes [8]–[33], where $\kappa-$symmetry is replaced by the more fundamental

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worldvolume superdiffeomorphism symmetry. These worldvolume superfield actions upon integration over the worldvolume Grassmann variables and the elimination of auxiliary fields by means of the equations of motion and gauge transformations have to reduce to the Green-Schwarz-type actions, where $\kappa-$symmetry appears as a remnant of the local worldvolume superdiffeomorphism symmetry. Further gauge fixing of the local symmetries for these Green-Schwarz-type actions yields the theory that is invariant under worldvolume global supersymmetry. So the role of $\kappa-$symmetry is to assemble local symmetries gauge fixed bosonic and fermionic degrees of freedom of the brane into the worldvolume supermultiplets. This is well known bose-fermi matching. Taken as the key principle for constructing superbranes it allows to define possible values for p and D (dimension of the target superspace), or, the brane scan. However, it turns out that for some types of branes like type-II superstrings in $D = 4, 6, 10$ [28, 34], $D = 11$ supermembrane [34], M5-brane [35] the construction of the worldvolume superfield actions with $\kappa-$symmetry being entirely replaced by the worldvolume superdiffeomorphisms encounters significant difficulties just in the point of the relation to the corresponding Green-Schwarz-type actions since the present in the doubly supersymmetric formulations auxiliary fields become dynamical.

The systematical study of doubly supersymmetric brane formulations is achieved in the framework of the superembedding approach (for reviews see [36, 37]). This is the supersymmetric generalization of the theory of surface embeddings [38] and deals with a brane superworldvolume embedded into a target superspace. Initially the idea of the consideration of bosonic strings as surfaces embedded into the target space was put forward in [39] (see also [40]). The relation to the gauge theories on the worldvolume was studied in [41]. The first indications of the efficiency of embedding approach in the supersymmetric theories of particles and strings were discovered in [3, 4, 5]. Further developments of superembedding approach and geometrodynamical condition as its basic ingredient were made in [24]. Detailed analysis of the case when the number of the superworldvolume Grassmann directions $n_{\rm wv}$ equals half the number of the target superspace Grassmann coordinates $n_{\rm ts}$ was performed in [34].

In supersymmetric theory of embeddings there appears the interesting phenomenon that has no analogs in the classical bosonic theory, namely, for certain branes geometrodynamical equation contains superbrane equations of motion. This leads to the new class of on-shell embeddings that are minimal from the beginning. It is precisely for the branes of this class one faces the described above obstacles when trying to construct worldvolume superfield models. (It is possible, however, to construct worldvolume superfield actions for the gauge fixed physical degrees of freedom in the framework of the method of nonlinear realizations both for off-shell and on-shell superbranes. The examples considered in the literature include codimension zero and one cases (see [42] and references therein). Superembedding approach treatment for these superbranes was performed e.g. in [32, 33, 43].) On the other hand, for on-shell embeddings the basic superembedding equation encompasses full information about the superbrane dynamics. If the superembedding is off-shell then the superbrane equations of motion follow from the requirement of the minimality of embedding. The situation is similar to that in the bosonic theory [38]–[41]. Then one is able to construct the doubly supersymmetric superfield actions for corresponding superbranes. This scheme was realized for various superparticle models [4, 5] and [8]–[16], null-superstrings [17], heterotic superstrings [18]–[27], $D = 3$ type-II superstring [28] (see also [29]), $D = 4$ $N = 1$ supermembrane [30, 31] and the space-filling branes in $D = 3, 4$ [32, 33]. However, proposed worldvolume superfield actions possess infinitely reducible superfield gauge symmetries that cause significant difficulties when trying to quantize these models. Alternative approach for constructing superbrane worldvolume superfield actions is provided by the generalized action principle [44]–[48].

The characteristic feature of the superembeddings considered so far is that the number of the superworldvolume Grassmann directions $n_{\rm wv}$ equals to the half of the number of target superspace Grassmann directions $n_{\rm ts}$. This situation corresponds to the most complete description of superbranes, where $\kappa-$symmetry of component formulations is entirely replaced by the local worldvolume supersymmetry. However, it is possible to contemplate more general situation with $n_{\rm wv} < n_{\rm ts}/2$ (the case $n_{\rm wv} > n_{\rm ts}/2$ seems to be too restrictive although there exist (super)particle models [49] with more than $n_{\rm ts}/2$ $\kappa-$symmetries (see also [50])) in an attempt to extend the class of off-shell superembeddings for which there exist worldvolume superfield formulations. Considered in the literature worldvolume superfield actions for particles and strings with $n_{\rm wv} < n_{\rm ts}/2$ [3, 4, 5, 18, 19, 21, 22, 23, 29, 51], for which $\kappa-$symmetries that were not realized as worldvolume superdiffeomorphisms appear as the local symmetries on the superfield level, suggest that such embeddings can still describe breaking of the half of target space supersymmetries, however, there can also exist configurations that break larger fraction of supersymmetry. It therefore seems to be instructive to study such embeddings with $n_{\rm wv} < n_{\rm ts}/2$ in the framework of the superembedding approach. In the present paper we consider $D = 10$ type-II superstrings as the representatives of on-shell (when $n_{\rm wv} = n_{\rm ts}/2$) superembeddings. We concentrate on $n = (1, 1)$ superworldsheet embedded into the flat $D = 10$ type-II target superspace that is of the most interest for establishing the relation with the NSR string [3, 4, 5, 52, 53, 54] invariant under the local $n = (1, 1)$ worldsheet supersymmetry.

2. Basic equations

The necessary ingredients of our construction are vector and spinor Lorentz harmonic variables [55]–[64] $(u_{\underline{m}}^{\underline{(n)}}, v_{\underline{\mu}}^{\underline{(\alpha)}})$ and their inverse $(u^{\underline{m}}_{\underline{(n)}}, v^{\underline{\mu}}_{\underline{(\alpha)}})$. For the $D = 10$ target superspace these are 10×10 and 16×16 orthonormal matrices respectively. Spinor harmonics also should satisfy certain harmonicity conditions that reduce the number of independent degrees of freedom of the spinor harmonics to the dimension of SO$(1, 9)$ Lorentz group equal to 45. In the presence of the superstring SO$(1, 9)_R$ group acting on the index in brackets of harmonics is broken down to SO$(1, 1) \times$ SO(8), thus the Lorentz harmonics adapted for the description of superstrings acquire the form

$$u_{\underline{m}}^{\underline{(n)}} = \left(u_{\underline{m}}^{+2}, u_{\underline{m}}^{i}, u_{\underline{m}}^{-2}\right), \qquad v_{\underline{\mu}}^{\underline{(\alpha)}} = \left(v^{+}_{\underline{\mu}q}, v^{-}_{\underline{\mu}\dot{q}}\right), \qquad v^{\underline{\mu}}_{\underline{(\alpha)}} \equiv \left(v_{\underline{\mu}}^{\underline{(\alpha)}}\right)^{-1} = \left(v_q{}^{-\underline{\mu}}, v_{\dot{q}}{}^{+\underline{\mu}}\right) \tag{2.1}$$

in the light-like notations for SO$(1, 1)$ that is identified with the worldsheet structure group. Indices $i, q, \dot{q}$ corespond to v, c, s representations of SO(8). Lorentz harmonics (2.1) parametrize the coset-space SO$(1, 9)/SO(1, 1) \times SO(8)$. Lorentz connection 1-form looks like

$$\Omega^{(\underline{n})(\underline{k})} = u_{\underline{m}}^{(\underline{n})} du^{\underline{m}(\underline{k})} : \qquad \Omega^{(0)} = \frac{1}{2} u_{\underline{m}}^{+2} du^{\underline{m}-2} ,$$
$$\Omega^{\pm 2i} = u_{\underline{m}}^{\pm 2} du^{\underline{m}i} , \qquad \Omega^{ij} = u_{\underline{m}}^{i} du^{\underline{m}j} . \tag{2.2}$$

In this set up $D = 10$ type-II target superspace coordinates can be splitted as follows

$$\begin{aligned} X^{(\underline{n})} &= X^{\underline{m}} u_{\underline{m}}^{(\underline{n})} = \left(x^{+2}, x^{i}, x^{-2}\right) , \\ \Theta^{1(\underline{\alpha})} &= \Theta^{1\underline{\mu}} v_{\underline{\mu}}^{(\underline{\alpha})} = \left(\theta_q^{1+}, \theta_{\dot q}^{1-}\right) , \\ \Theta^{2}_{(\underline{\beta})} &= \Theta^{2}_{\underline{\mu}} v^{\underline{\mu}}_{(\underline{\beta})} = \left(\theta_q^{2-}, \theta_{\dot q}^{2+}\right) \end{aligned} \tag{2.3}$$

for the type-IIA superspace and

$$X^{(\underline{n})} = X^{\underline{m}} u_{\underline{m}}^{(\underline{n})} = \left(x^{+2}, x^{i}, x^{-2}\right) , \qquad \Theta^{1,2\ (\underline{\alpha})} = \Theta^{1,2\ \underline{\mu}} v_{\underline{\mu}}^{(\underline{\alpha})} = \left(\theta_q^{1,2+}, \theta_{\dot q}^{1,2-}\right) \tag{2.4}$$

for the type-IIB superspace.

Basic 1-forms of the type-IIA target superspace then acquire the form

$$\Pi^{\underline{m}} = dX^{\underline{m}} - id\Theta^{1\underline{\mu}} \sigma^{\underline{m}}_{\underline{\mu}\underline{\nu}} \Theta^{1\underline{\nu}} - id\Theta^{2}_{\underline{\mu}} \tilde\sigma^{\underline{m}\underline{\mu}\underline{\nu}} \Theta^{2}_{\underline{\nu}} = \frac{1}{2} u^{\underline{m}+2} \Pi^{-2} + \frac{1}{2} u^{\underline{m}-2} \Pi^{+2} - u^{\underline{m}\ i} \Pi^{i} , \tag{2.5}$$

$$d\Theta^{1\underline{\mu}} = v^{\underline{\mu}}_{(\underline{\alpha})} \pi^{1(\underline{\alpha})} = v_q^{-\underline{\mu}} \pi_q^{1+} + v_{\dot q}^{+\underline{\mu}} \pi_{\dot q}^{1-} , \qquad d\Theta^{2}_{\underline{\mu}} = v_{\underline{\mu}}^{(\underline{\alpha})} \pi^{2}_{(\underline{\alpha})} = v^{+}_{\underline{\mu}q} \pi_q^{2-} + v^{-}_{\underline{\mu}\dot q} \pi_{\dot q}^{2+} , \tag{2.6}$$

where

$$\begin{aligned} \Pi^{+2} &= \nabla x^{+2} - \Omega^{+2i} x^{i} - 2i\pi_q^{1+} \theta_q^{1+} - 2i\pi_{\dot q}^{2+} \theta_{\dot q}^{2+} , \\ \Pi^{-2} &= \nabla x^{-2} - \Omega^{-2i} x^{i} - 2i\pi_{\dot q}^{1-} \theta_{\dot q}^{1-} - 2i\pi_q^{2-} \theta_q^{2-} , \end{aligned} \tag{2.7}$$

$$\begin{aligned} \Pi^{i} &= \nabla x^{i} - \frac{1}{2}\Omega^{+2i} x^{-2} - \frac{1}{2}\Omega^{-2i} x^{+2} - i\pi_q^{1+} \gamma^{i}_{q\dot q} \theta_{\dot q}^{1-} - \\ &\quad - i\pi_{\dot q}^{1-} \tilde\gamma^{i}_{\dot q q} \theta_q^{1+} + i\pi_q^{2-} \gamma^{i}_{q\dot q} \theta_{\dot q}^{2+} + i\pi_{\dot q}^{2+} \tilde\gamma^{i}_{\dot q q} \theta_q^{2-} , \end{aligned} \tag{2.8}$$

$$\pi^{1(\underline{\alpha})} = \begin{pmatrix} \pi_q^{1+} \\ \pi_{\dot q}^{1-} \end{pmatrix} = \begin{pmatrix} \nabla\theta_q^{1+} - \frac{1}{2}\gamma^{i}_{q\dot r}\theta_{\dot r}^{1-}\Omega^{+2i} \\ \nabla\theta_{\dot q}^{1-} - \frac{1}{2}\tilde\gamma^{i}_{\dot q r}\theta_r^{1+}\Omega^{-2i} \end{pmatrix} , \tag{2.9}$$

$$\pi^{2}_{(\underline{\alpha})} = \begin{pmatrix} \pi_q^{2-} \\ \pi_{\dot q}^{2+} \end{pmatrix} = \begin{pmatrix} \nabla\theta_q^{2-} + \frac{1}{2}\gamma^{i}_{q\dot r}\theta_{\dot r}^{2+}\Omega^{-2i} \\ \nabla\theta_{\dot q}^{2+} + \frac{1}{2}\tilde\gamma^{i}_{\dot q r}\theta_r^{2-}\Omega^{+2i} \end{pmatrix} . \tag{2.10}$$

In (2.7)–(2.10) we introduced covariant differentials ∇ defined as

$$\nabla x^{\pm 2} = dx^{\pm 2} \pm \Omega^{(0)} x^{\pm 2} , \qquad \nabla x^{i} = dx^{i} - \Omega^{ij} x^{j} ,$$
$$\nabla\theta_q^{\pm} = d\theta_q^{\pm} \pm \frac{1}{2}\Omega^{(0)}\theta_q^{\pm} - \frac{1}{4}\Omega_{qp}\theta_p^{\pm} , \qquad \nabla\theta_{\dot q}^{\pm} = d\theta_{\dot q}^{\pm} \pm \frac{1}{2}\Omega^{(0)}\theta_{\dot q}^{\pm} - \frac{1}{4}\Omega_{\dot q\dot p}\theta_{\dot p}^{\pm} , \tag{2.11}$$

where $\Omega_{qp} \equiv \gamma^{ij}_{qp}\Omega^{ij}$ and $\Omega_{\dot q\dot p} \equiv \tilde\gamma^{ij}_{\dot q\dot p}\Omega^{ij}$.

Analogously, in case of the type-IIB target superspace one has

$$\begin{aligned} \Pi^{+2} &= \nabla x^{+2} - \Omega^{+2i} x^{i} - 2i\pi_q^{1+} \theta_q^{1+} - 2i\pi_q^{2+} \theta_q^{2+} , \\ \Pi^{-2} &= \nabla x^{-2} - \Omega^{-2i} x^{i} - 2i\pi_{\dot q}^{1-} \theta_{\dot q}^{1-} - 2i\pi_{\dot q}^{2-} \theta_{\dot q}^{2-} , \end{aligned} \tag{2.12}$$

$$\Pi^{i} = \nabla x^{i} - \frac{1}{2}\Omega^{+2i} x^{-2} - \frac{1}{2}\Omega^{-2i} x^{+2} - i\pi_q^{1+} \gamma^{i}_{q\dot q} \theta_{\dot q}^{1-} - i\pi_{\dot q}^{1-} \tilde\gamma^{i}_{\dot q q} \theta_q^{1+} + (1 \leftrightarrow 2) . \tag{2.13}$$

The definitions of fermionic 1-forms $\pi^{1,2(\underline{\alpha})}$ coincide with (2.9).

The integrability conditions for 1-forms $\Pi^{\underline{m}}$ and $d\Theta$ that can be considered as Maurer-Cartan equations for the supertranslations in the flat superspace [65] in the new string basis defined by (2.3) and (2.4) transform into

$$\nabla\Pi^{(\underline{m})} = -i\pi^{1(\underline{\alpha})}\sigma^{(\underline{m})}_{(\underline{\alpha})(\underline{\beta})}\pi^{1(\underline{\beta})} - i\pi^{2}_{(\underline{\alpha})}\tilde{\sigma}^{(\underline{m})(\underline{\alpha})(\underline{\beta})}\pi^{2}_{(\underline{\beta})}\,, \qquad (IIA) \tag{2.14}$$

$$\nabla\Pi^{(\underline{m})} = -i\pi^{1(\underline{\alpha})}\sigma^{(\underline{m})}_{(\underline{\alpha})(\underline{\beta})}\pi^{1(\underline{\beta})} - (1 \leftrightarrow 2)\,, \qquad (IIB) \tag{2.15}$$

$$\nabla\pi^{(\underline{\alpha})} = d\pi^{(\underline{\alpha})} - \frac{1}{4}\pi^{(\underline{\beta})}\Omega_{(\underline{\beta})}{}^{(\underline{\alpha})} = 0\,, \qquad \nabla\pi_{(\underline{\alpha})} = d\pi_{(\underline{\alpha})} - \frac{1}{4}\pi_{(\underline{\beta})}\Omega^{(\underline{\beta})}{}_{(\underline{\alpha})} = 0\,. \tag{2.16}$$

The worldsheet superspace is locally parametrized by the coordinates $z^M = (\xi^m, \eta^{\mu q})$. ξ^m is the $2d$ vector and $\eta^{\mu q}$ is the $2d$ Majorana spinor containing two components with opposite chiralities, index $q = 1, \ldots, n$ labels extended worldsheet supersymmetries. The $2d$ superspaces of that kind are referred to as (n, n) superspaces. For the $D = 10$ type-II target superspace n can take values from 1 to 8. $n = (8, 8)$ Superzweinbein 1-forms in the light-like notations for $\mathrm{SO}(1,1)$ group are the following

$$e^{\pm 2}\,, \qquad e^{+}_{q}\,, \qquad e^{-}_{q} \qquad \text{or} \qquad e^{\pm 2}\,, \qquad e^{+}_{q}\,, \qquad e^{-}_{\dot{q}}\,. \tag{2.17}$$

The first choice is adapted for the description of superworldsheet embedding into the type-IIA target superspace, the second choice is adapted for the type-IIB target superspace. The motivation for this is the structure of irreducible $\kappa-$symmetry transformations of the corresponding component formulations [62].

In the string basis for the target superspace coordinates (2.3) and (2.4) bosonic embedding equations have the well known form

$$\Pi^{+2} = e^{+2}\,, \qquad \Pi^{-2} = e^{-2}, \qquad \Pi^{i} = 0\,. \tag{2.18}$$

They state that it is always possible by means of proper rotations to equate two bosonic components of the target space supervielbein to the bosonic components of the worldsheet superzweinbein, the rest of the target space supervielbein bosonic components being orthogonal to the string. What concerns the fermionic embedding equations there exist two different cases. If $n = 8$ one is able to choose

$$\pi^{1+}_{q} = e^{+}_{q}\,, \qquad \pi^{2-}_{q} = e^{-}_{q} \tag{2.19}$$

in the type-IIA case and

$$\pi^{1+}_{q} = e^{+}_{q}\,, \qquad \pi^{2-}_{\dot{q}} = e^{-}_{\dot{q}} \tag{2.20}$$

in the type-IIB case. When $n < 8$ we are able to introduce additional bosonic $\mathrm{SO}(8)$ spinors $\lambda^{a}_{q}(z^M)$, $\lambda^{\dot{a}}_{\dot{q}}(z^M)$ in order to adjust covariantly Grassmann components of the target space supervielbein to the Grassmann components of the superzweinbein. For establishing a relation between the NSR string and the type-II GS superstring models the embedding of the $n = (1,1)$ worldsheet superspace is of the primary importance. In this case we have

$$\pi^{1+}_{q}\lambda^{1}_{q} = e^{+}\,, \qquad \pi^{2-}_{q}\lambda^{2}_{q} = e^{-} \tag{2.21}$$

for the type-IIA target superspace and

$$\pi_q^{1+}\lambda_q^1 = e^+ , \qquad \pi_{\dot q}^{2-}\lambda_{\dot q}^2 = e^- \tag{2.22}$$

for the type-IIB target superspace.

Target superspace fermionic 1-forms entering (2.21) and (2.22) have the following general decompositions over the $n = (1,1)$ superzweinbein basis

$$\pi_q^{1+} = e^{+2}\psi_{+2q}^{1+} + e^{-2}\psi_{-2q}^{1+} + e^+\chi_q^1 + e^- h_{-q}^{1+} , \tag{2.23}$$

$$\pi_q^{2-} = e^{+2}\psi_{+2q}^{2-} + e^{-2}\psi_{-2q}^{2-} + e^+ h_{+q}^{2-} + e^-\chi_q^2 \tag{2.24}$$

in the type-IIA case, (2.23) and

$$\pi_{\dot q}^{2-} = e^{+2}\psi_{+2\dot q}^{2-} + e^{-2}\psi_{-2\dot q}^{2-} + e^+ h_{+\dot q}^{2-} + e^-\chi_{\dot q}^2 \tag{2.25}$$

in the type-IIB case.

Other target superspace fermionic 1-forms decompositions read

$$\pi_{\dot q}^{1-} = e^{+2}\psi_{+2\dot q}^{1-} + e^{-2}\psi_{-2\dot q}^{1-} + e^+ h_{+\dot q}^{1-} + e^-\chi_{\dot q}^1 , \tag{2.26}$$

$$\pi_{\dot q}^{2+} = e^{+2}\psi_{+2\dot q}^{2+} + e^{-2}\psi_{-2\dot q}^{2+} + e^+\chi_{\dot q}^2 + e^- h_{-\dot q}^{2+} \tag{2.27}$$

for the type-IIA target superspace, (2.26) and

$$\pi_q^{2+} = e^{+2}\psi_{+2q}^{2+} + e^{-2}\psi_{-2q}^{2+} + e^+\chi_q^2 + e^- h_{-q}^{2+} , \tag{2.28}$$

for the type-IIB target superspace. In the decompositions (2.23)–(2.28) $\psi(z^M)$-superfields are Grassmann ones, other superfields are bosonic. Thus, relations (2.21) and (2.22) impose certain restrictions on the decomposition coefficients (2.23)–(2.25). Namely, from (2.21) we deduce

$$\psi_{\pm 2q}^{1+}\lambda_q^1 = h_{-q}^{1+}\lambda_q^1 = 0 , \qquad \chi_q^1\lambda_q^1 = 1 , \tag{2.29}$$

$$\psi_{\pm 2q}^{2-}\lambda_q^2 = h_{+q}^{2-}\lambda_q^2 = 0 , \qquad \chi_q^2\lambda_q^2 = 1 \tag{2.30}$$

and from (2.22) we obtain (2.29) and

$$\psi_{\pm 2\dot q}^{2-}\lambda_{\dot q}^2 = h_{+\dot q}^{2-}\lambda_{\dot q}^2 = 0 , \qquad \chi_{\dot q}^2\lambda_{\dot q}^2 = 1 . \tag{2.31}$$

Further analysis of embedding lies in the investigation of integrability conditions for the basic superembedding equations in each case together with Maurer-Cartan equations that for the case of strings embedding acquire the following form

$$d\Omega^{(0)} - \frac{1}{2}\Omega^{+2i}\Omega^{-2i} = 0 , \tag{2.32}$$

$$d\Omega^{\pm 2i} \mp \Omega^{(0)}\Omega^{\pm 2i} + \Omega^{ij}\Omega^{\pm 2j} = 0 , \tag{2.33}$$

$$d\Omega^{ij} + \Omega^{ik}\Omega^{kj} + \frac{1}{2}\Omega^{+2i}\Omega^{-2j} - \frac{1}{2}\Omega^{-2i}\Omega^{+2j} = 0 . \tag{2.34}$$

Equations (2.32)–(2.34) are known as Gauss, Codazzi and Ricci equations correspondingly.

In the next section we will examine the embedding of $n = (8,8)$ worldsheet superspace into the $D = 10$ type-IIA target superspace. This problem was considered before [34], we, however, turn to it again both for the completeness of presentation and for the comparison with the $n = (1,1)$ superworldsheet embedding (section 4). In sections 5 and 6 we address the same problems for the target superspace of the type-IIB.

3. Embedding of the $n=(8,8)$ superworldsheet into $D=10$ $N=2A$ target superspace

This case is described by superembedding equations (2.18) and (2.19). We begin with deriving their integrability conditions. Consider first the differentials of bosonic embedding equations (2.18)

$$d\Pi^{+2} = de^{+2} , \qquad d\Pi^{-2} = de^{-2} , \qquad d\Pi^{i} = 0 . \tag{3.1}$$

Splitting (2.14) into the tangent and orthogonal to the worldsheet parts, defining torsion 2-form for the worldsheet superspace under consideration

$$T^{\pm 2} = de^{\pm 2} \mp \omega^{(0)} e^{\pm 2} , \qquad T_q^{\pm} = de_q^{\pm} \mp \frac{1}{2}\omega^{(0)} e_q^{\pm} + \frac{1}{4}\omega_{qp} e_p^{\pm} , \tag{3.2}$$

where $\omega^{(0)}$ and ω^{qp} are intrinsic connection 1-forms, and plugging all that back into (3.1) one finds

$$\Omega^{(0)} e^{+2} - 2ie_q^{+} e_q^{+} - 2i\pi_{\dot q}^{2+}\pi_{\dot q}^{2+} = T^{+2} + \omega^{(0)} e^{+2} , \tag{3.3}$$

$$-\Omega^{(0)} e^{-2} - 2i\pi_{\dot q}^{1-}\pi_{\dot q}^{1-} - 2ie_q^{-} e_q^{-} = T^{-2} - \omega^{(0)} e^{+2} , \tag{3.4}$$

$$-\frac{1}{2}\Omega^{+2i} e^{-2} - \frac{1}{2}\Omega^{-2i} e^{+2} - 2ie_q^{+}\gamma_{q\dot q}^{i}\pi_{\dot q}^{1-} + 2ie_q^{-}\gamma_{q\dot q}^{i}\pi_{\dot q}^{2+} = 0 . \tag{3.5}$$

It is possible to identify induced and intrinsic SO$(1,1)$ connections by setting

$$\Omega^{(0)} = \omega^{(0)} . \tag{3.6}$$

(Another possible way of consideration is to impose supergravity constraints on the intrinsic torsion 2-form and to introduce modified connections $\Omega(D) = \Omega(d) - \omega(d)$. As was shown in [34] for the case of strings $\Omega(D) = 0$, so induced and intrinsic connections coincide.) Thus, $+2$ and -2 torsion components acquire the form

$$T^{+2} = -2ie_q^{+} e_q^{+} - 2i\pi_{\dot q}^{2+}\pi_{\dot q}^{2+} , \qquad T^{-2} = -2i\pi_{\dot q}^{1-}\pi_{\dot q}^{1-} - 2ie_q^{-} e_q^{-} . \tag{3.7}$$

Application of differential operation to the fermionic embedding equations (2.19) and the utilization of (3.2) together with (2.16) leads to

$$\begin{aligned} -\frac{1}{4}\Omega_{qp} e_p^{+} - \frac{1}{2}\gamma_{q\dot p}^{i}\Omega^{+2i}\pi_{\dot p}^{1-} &= T_q^{+} - \frac{1}{4}\omega_{qp} e_p^{+} , \\ -\frac{1}{4}\Omega_{qp} e_p^{-} + \frac{1}{2}\gamma_{q\dot p}^{i}\Omega^{-2i}\pi_{\dot p}^{2+} &= T_q^{-} - \frac{1}{4}\omega_{qp} e_p^{-} . \end{aligned} \tag{3.8}$$

We are able to identify induced and intrinsic SO(8) connection 1-forms

$$\Omega_{qp} = \omega_{qp} \tag{3.9}$$

to obtain in this way

$$T_q^{+} = -\frac{1}{2}\gamma_{q\dot p}^{i}\Omega^{+2i}\pi_{\dot p}^{1-} , \qquad T_q^{-} = \frac{1}{2}\gamma_{q\dot p}^{i}\Omega^{-2i}\pi_{\dot p}^{2+} . \tag{3.10}$$

Then we should explore the integrability conditions for those target space supervielbein fermionic components that were not identified with the superzweinbein fermionic components, i.e. for $\pi^{1-}_{\dot q}$, $\pi^{2+}_{\dot q}$. On one hand, we have from (2.16)

$$\begin{aligned} d\pi^{1-}_{\dot q} &= -\frac{1}{2}\Omega^{(0)}\pi^{1-}_{\dot q} - \frac{1}{4}\Omega_{\dot q\dot p}\pi^{1-}_{\dot p} - \frac{1}{2}\tilde\gamma^i_{\dot q p}\Omega^{-2i}\pi^{1+}_p\,, \\ d\pi^{2+}_{\dot q} &= \frac{1}{2}\Omega^{(0)}\pi^{2+}_{\dot q} - \frac{1}{4}\Omega_{\dot q\dot p}\pi^{2+}_{\dot p} + \frac{1}{2}\tilde\gamma^i_{\dot q p}\Omega^{+2i}\pi^{2-}_p \end{aligned} \tag{3.11}$$

but, on the other hand, using the decompositions of $\pi^{1-}_{\dot q}$ and $\pi^{2+}_{\dot q}$ over the superworldsheet basis

$$\begin{aligned} \pi^{1-}_{\dot q} &= e^{+2}\psi^{1-}_{+2\dot q} + e^{-2}\psi^{1-}_{-2\dot q} + e^+_q h^{1-}_{+q\dot q} + e^-_q\chi^1_{q\dot q}\,, \\ \pi^{2+}_{\dot q} &= e^{+2}\psi^{2+}_{+2\dot q} + e^{-2}\psi^{2+}_{-2\dot q} + e^+_q\chi^2_{q\dot q} + e^-_q h^{2+}_{-q\dot q} \end{aligned} \tag{3.12}$$

we derive

$$\begin{aligned} &(e^{+2}\nabla + T^{+2})\psi^{1-}_{+2\dot q} + (e^{-2}\nabla + T^{-2})\psi^{1-}_{-2\dot q} + (e^+_p\nabla + T^+_p)h^{1-}_{+p\dot q} + (e^-_p\nabla + T^-_p)\chi^1_{p\dot q} = \\ &\quad = \frac{1}{2}e^+_p\gamma^i_{p\dot q}\Omega^{-2i}\,, \end{aligned} \tag{3.13}$$

$$\begin{aligned} &(e^{+2}\nabla + T^{+2})\psi^{2+}_{+2\dot q} + (e^{-2}\nabla + T^{-2})\psi^{2+}_{-2\dot q} + (e^+_p\nabla + T^+_p)\chi^2_{p\dot q} + (e^-_p\nabla + T^-_p)h^{2+}_{-p\dot q} = \\ &\quad = -\frac{1}{2}e^-_p\gamma^i_{p\dot q}\Omega^{+2i}\,. \end{aligned} \tag{3.14}$$

So, we have found all the integrability conditions for the embedding under consideration and turn now to their analysis. Consider, first, eq. (3.5). Its spinor-spinor components read

$$\gamma^i_{q\dot q}h^{1-}_{+p\dot q} + \gamma^i_{p\dot q}h^{1-}_{+q\dot q} = 0\,, \qquad \gamma^i_{q\dot q}h^{2+}_{-p\dot q} + \gamma^i_{p\dot q}h^{2+}_{-q\dot q} = 0\,, \qquad -\gamma^i_{q\dot q}\chi^1_{p\dot q} + \gamma^i_{p\dot q}\chi^2_{q\dot q} = 0\,. \tag{3.15}$$

To analyse eqs. (3.15) it is necessary to decompose $h^{1-}_{+q\dot q}$, $h^{2+}_{-q\dot q}$, $\chi^{1,2}_{q\dot q}$ on the SO(8) irreducible parts

$$\begin{aligned} h^{1-}_{+q\dot q} &= \gamma^i_{q\dot q}h^{1-i}_+ + \gamma^{ijk}_{q\dot q}h^{1-ijk}_+\,, \\ h^{2+}_{-q\dot q} &= \gamma^i_{q\dot q}h^{2+i}_- + \gamma^{ijk}_{q\dot q}h^{2+ijk}_-\,, \\ \chi^{1,2}_{q\dot q} &= \gamma^i_{q\dot q}\chi^{1,2i} + \gamma^{ijk}_{q\dot q}\chi^{1,2ijk}\,. \end{aligned} \tag{3.16}$$

Substituting (3.16) back into (3.15) after some manipulations with SO(8) $\sigma-$matrices one obtains

$$h^{1-}_{+q\dot q} = h^{2+}_{-q\dot q} = \chi^{1,2i} = 0\,, \qquad \chi^{1ijk} = -\chi^{2ijk}\,. \tag{3.17}$$

Then using Bianchi identities for the induced supertorsion it is possible to show that

$$\chi^{1ijk} = 0\,. \tag{3.18}$$

Other components of (3.5) define the second fundamental form through the embedding functions (3.12)

$$\Omega^{+2i}_{+2} - \Omega^{-2i}_{-2} = 0\,, \qquad \frac{1}{2}\Omega^{\pm 2i}_{+q} + 2i\gamma^i_{q\dot q}\psi^{1-}_{\mp 2\dot q} = 0\,, \qquad -\frac{1}{2}\Omega^{\pm 2i}_{-q} + 2i\gamma^i_{q\dot q}\psi^{2+}_{\mp 2\dot q} = 0\,. \tag{3.19}$$

We are in a position to consider the consequences of the integrability conditions (3.13) and (3.14). From (3.14) after taking into account (3.17)–(3.19) we deduce

$$\begin{aligned} \Omega^{+2i}_{+q} &= 0 \Longrightarrow \psi^{1-}_{-2\dot q} = 0\,, \qquad \psi^{2+}_{+2\dot q} = 0 \Longrightarrow \Omega^{-2i}_{-q} = 0\,, \\ \psi^{2+}_{-2\dot q} &= -\frac{i}{32}\tilde\gamma^i_{\dot q q}\Omega^{+2i}_{-q}\,, \qquad (3.20) \\ \Omega^{+2i}_{+2} &= \Omega^{-2i}_{-2} = 0\,, \qquad \Omega^{+2i}_{-2} = \frac{1}{4}\nabla_{-p}\gamma^i_{p\dot q}\psi^{2+}_{-2\dot q}\,, \qquad (3.21) \\ \nabla_{+p}\psi^{2+}_{-2\dot q} &= 0\,, \qquad \nabla_{+2}\psi^{2+}_{-2\dot q} = 0\,. \qquad (3.22) \end{aligned}$$

From the integrability conditions for $\pi^{1-}_{\dot q}$ (3.13) there follow the relations

$$\begin{aligned} \psi^{1-}_{+2\dot q} &= \frac{i}{32}\tilde\gamma^i_{\dot q p}\Omega^{-2i}_{+p}\,, \qquad \Omega^{-2i}_{+2} = -\frac{1}{4}\nabla_{+q}\gamma^i_{q\dot q}\psi^{1-}_{+2\dot q}\,, \qquad (3.23) \\ \nabla_{-q}\psi^{1-}_{+2\dot q} &= 0\,, \qquad \nabla_{-2}\psi^{1-}_{+2\dot q} = 0\,. \qquad (3.24) \end{aligned}$$

We observe that from the variety of functions entering (3.12) there remained only two chiral superfields $\psi^{1-}_{+2\dot q}(z^M)$ and $\psi^{2+}_{-2\dot q}(z^M)$.

Maurer-Cartan equations (2.32), (2.33) and (2.34) in the considered case do not contain any additional dynamical information.

Supertorsion components after substitution of (3.17)–(3.18) and (3.20)–(3.24) reduce to

$$\begin{aligned} T^{\pm 2}_{\pm q \pm p} &= -4i\delta_{qp}\,, \qquad (3.25) \\ T^{+q}_{-p+2} &= -T^{-p}_{+q-2} = -2i\gamma^i_{q\dot q}\psi^{1-}_{+2\dot q}\gamma^i_{p\dot p}\psi^{2+}_{-2\dot p}\,, \qquad T^{\pm q}_{+2-2} = \pm\frac{i}{16}\nabla_{\mp p}T^{\pm q}_{\mp p \pm 2}\,. \qquad (3.26) \end{aligned}$$

Explicit expressions for the original target superspace supervielbeins follow from (2.5)–(2.6)

$$\begin{aligned} \Pi^{\underline m} &= u^{\underline m +2}e^{-2} + u^{\underline m -2}e^{+2}\,, \qquad (3.27) \\ d\Theta^{1\underline\mu} &= e^{+2}\psi^{1-}_{+2\dot q}v^{+\underline\mu}_{\dot q} + e^{+}_{q}v^{-\underline\mu}_{q}\,, \qquad d\Theta^{2}_{\underline\mu} = e^{-2}\psi^{2+}_{-2\dot q}v^{-}_{\underline\mu\dot q} + e^{-}_{q}v^{+}_{\underline\mu q}\,. \qquad (3.28) \end{aligned}$$

Let us show that the considered embedding is on-shell. The type-IIA superstring equations of motion in the twistor-like Lorentz harmonic formulation read

$$D_{-2}\theta^{1\underline\mu}(\xi)v^{-}_{\underline\mu\dot q} = 0\,, \qquad D_{+2}\theta^{2}_{\underline\mu}(\xi)v^{+\underline\mu}_{\dot q} = 0\,, \qquad (3.29)$$

$$\frac{1}{2}\partial_\mu\left(e\sum_{\pm}e^{\mu\pm 2}u^{\mp 2}_{\underline m}\right) - i\epsilon^{\mu\nu}\left(\partial_\mu\theta^{1\underline\mu}(\xi)\sigma_{\underline{m\mu\nu}}\partial_\nu\theta^{1\underline\nu}(\xi) - \partial_\mu\theta^2_{\underline\mu}(\xi)\tilde\sigma^{\underline{\mu\nu}}_{\underline m}\partial_\nu\theta^2_{\underline\nu}(\xi)\right) = 0\,. \qquad (3.30)$$

Upon utilization of (2.2) and (2.6) these equations can be written in terms of the second fundamental form and functions $\psi^{1+}_{\pm 2q}(\xi) = D_{\pm 2}\theta^{1\underline\mu}v^{+}_{\underline\mu q}$, $\psi^{1-}_{\pm 2\dot q}(\xi) = D_{\pm 2}\theta^{1\underline\mu}v^{-}_{\underline\mu\dot q}$, $\psi^{2+}_{\pm 2\dot q}(\xi) = D_{\pm 2}\theta^{2}_{\underline\mu}v^{+\underline\mu}_{\dot q}$, $\psi^{2-}_{\pm 2q}(\xi) = D_{\pm 2}\theta^2_{\underline\mu}v^{-\underline\mu}_{q}$ as

$$\begin{aligned} &\psi^{1-}_{-2\dot q} = \psi^{2+}_{+2\dot q} = 0\,, \qquad (3.31) \\ &\Omega^{+2i}_{+2} + \Omega^{-2i}_{-2} + 4i\left(\psi^{1+}_{+2q}\gamma^i_{q\dot q}\psi^{1-}_{-2\dot q} - \psi^{1+}_{-2q}\gamma^i_{q\dot q}\psi^{1-}_{+2\dot q} + \psi^{2-}_{+2q}\gamma^i_{q\dot q}\psi^{2+}_{-2\dot q} - \psi^{2-}_{-2q}\gamma^i_{q\dot q}\psi^{2+}_{+2\dot q}\right) = \\ &\quad = 0\,. \qquad (3.32) \end{aligned}$$

Superstring equations of motion in the form (3.31) and (3.32) permit natural worldsheet superfield generalization. Then noting that from the fermionic embedding equations (2.19) it follows that $\psi^{1+}_{\pm 2q}(z^M) = \psi^{2-}_{\pm 2q}(z^M) = 0$ we find that (3.32) acquires the form

$$\Omega^{+2i}_{+2}(z^M) + \Omega^{-2i}_{-2}(z^M) = 0\,. \tag{3.33}$$

It is nothing but the minimal embedding condition known from the bosonic theory. On the other hand, (3.33) is satisfied by eqs. (3.21) that stem from the integrability conditions for superembedding equations.

4. Embedding of the $n = (1,1)$ superworldsheet into $D = 10$ $N = 2A$ target superspace

Following the guidelines of the previous section first we have to derive integrability conditions for the basic superembedding equations (2.18) and (2.21). Applying external differential to them and using the definition of the supertorsion 2-form for the $n = (1,1)$ worldsheet superspace

$$T^{\pm 2} = de^{\pm 2} \mp \omega^{(0)} e^{\pm 2}\,, \qquad T^{\pm} = de^{\pm} \mp \frac{1}{2}\omega^{(0)} e^{\pm} \tag{4.1}$$

we derive

$$T^{+2} = -2i\pi^{1+}_q \pi^{1+}_q - 2i\pi^{2+}_{\dot q}\pi^{2+}_{\dot q}\,, \qquad T^{-2} = -2i\pi^{1-}_{\dot q}\pi^{1-}_{\dot q} - 2i\pi^{2-}_q \pi^{2-}_q\,, \tag{4.2}$$

$$T^{+} = \pi^{1+}_q \nabla\lambda^1_q - \frac{1}{2}\lambda^1_q \gamma^i_{q\dot q}\Omega^{+2i}\pi^{1-}_{\dot q}\,, \qquad T^{-} = \pi^{2-}_q \nabla\lambda^2_q + \frac{1}{2}\lambda^2_q \gamma^i_{q\dot q}\Omega^{-2i}\pi^{2+}_{\dot q}\,, \tag{4.3}$$

$$-\frac{1}{2}\Omega^{+2i}e^{-2} - \frac{1}{2}\Omega^{-2i}e^{+2} - 2i\pi^{1+}_q \gamma^i_{q\dot q}\pi^{1-}_{\dot q} + 2i\pi^{2-}_q \gamma^i_{q\dot q}\pi^{2+}_{\dot q} = 0\,. \tag{4.4}$$

Like in section 2 we identified $SO(1,1)$ induced and intrinsic connections

$$\Omega^{(0)} = \omega^{(0)}\,, \tag{4.5}$$

and also introduced the covariant differential for the auxiliary spinors $\lambda^{1,2}_q(z^M)$ as

$$\nabla\lambda^{1,2}_q = d\lambda^{1,2}_q - \frac{1}{4}\Omega_{qp}\lambda^{1,2}_p\,. \tag{4.6}$$

The integrability conditions for π^{1+}_q, π^{2-}_q and $\pi^{1-}_{\dot q}, \pi^{2+}_{\dot q}$ components of the target space supervielbein have the form

$$\begin{aligned}
&(e^{+2}\nabla + T^{+2})\psi^{1+}_{+2q} + (e^{-2}\nabla + T^{-2})\psi^{1+}_{-2q} + (e^{+}\nabla + T^{+})\chi^1_q + (e^{-}\nabla + T^{-})h^{1+}_{-q} = \\
&\quad = -\frac{1}{2}\gamma^i_{q\dot p}\Omega^{+2i}\pi^{1-}_{\dot p}\,, &(4.7)\\
&(e^{+2}\nabla + T^{+2})\psi^{2-}_{+2q} + (e^{-2}\nabla + T^{-2})\psi^{2-}_{-2q} + (e^{+}\nabla + T^{+})h^{2-}_{+q} + (e^{-}\nabla + T^{-})\chi^2_q = \\
&\quad = \frac{1}{2}\gamma^i_{q\dot p}\Omega^{-2i}\pi^{2+}_{\dot p}\,, &(4.8)\\
&(e^{+2}\nabla + T^{+2})\psi^{1-}_{+2\dot q} + (e^{-2}\nabla + T^{-2})\psi^{1-}_{-2\dot q} + (e^{+}\nabla + T^{+})h^{1-}_{+\dot q} + (e^{-}\nabla + T^{-})\chi^1_{\dot q} = \\
&\quad = -\frac{1}{2}\tilde\gamma^i_{\dot q p}\Omega^{-2i}\pi^{1+}_p\,, &(4.9)\\
&(e^{+2}\nabla + T^{+2})\psi^{2+}_{+2\dot q} + (e^{-2}\nabla + T^{-2})\psi^{2+}_{-2\dot q} + (e^{+}\nabla + T^{+})\chi^2_{\dot q} + (e^{-}\nabla + T^{-})h^{2+}_{-\dot q} = \\
&\quad = \frac{1}{2}\tilde\gamma^i_{\dot q p}\Omega^{+2i}\pi^{2-}_p\,. &(4.10)
\end{aligned}$$

Note, that the multiplication of (4.7) by λ_q^1 and of (4.8) by λ_q^2 produces expressions for the induced supertorsion $\pm$ components (4.3) provided the integrability conditions for (2.29) and (2.30) are used.

The expressions for the induced worldsheet supertorsion (4.2)–(4.3) do not yield $2d$ $n=(1,1)$ supergravity constraints in contrast with the $n=(8,8)$ case, so let us contemplate the consequences of their imposition. Conventional $2d$ $n=(1,1)$ supergravity constraints have the form [66]

$$T^{\pm 2}_{\pm\pm} = -4i,\ T^{+2}_{\pm -} = T^{-2}_{\pm +} = 0\,, \qquad T^{\pm 2}_{+2-2} = 0\,, \qquad T^{\pm}_{\pm\pm} = T^{+}_{\pm -} = T^{-}_{\pm +} = 0\,. \tag{4.11}$$

The consideration of Bianchi identities allows to represent all non-zero supertorsion components through the single scalar superfield $S(z^M)$ and its derivatives

$$T^{+}_{-+2} = S,\ T^{-}_{+-2} = -S\,, \qquad T^{\pm}_{+2-2} = \mp\frac{i}{2}\nabla_{\mp}S\,. \tag{4.12}$$

Below we will adduce explicit expression for S in terms of the second fundamental form.

The insertion of (4.11) and (4.12) into (4.2) and (4.3) results in the following algebraic equations

$$h^{1+}_{-q} = h^{2-}_{+q} = 0\,, \qquad h^{1-}_{+\dot q} = h^{2+}_{-\dot q} = 0\,, \qquad \chi^1_q\chi^1_q + \chi^2_{\dot q}\chi^2_{\dot q} = 1\,, \qquad \chi^2_q\chi^2_q + \chi^1_{\dot q}\chi^1_{\dot q} = 1\,. \tag{4.13}$$

Then from the integrability conditions (4.7)–(4.10) one infers as the spinor-spinor component equations

$$\psi^{1+}_{+2q} = -\frac{i}{2}\nabla_{+}\chi^1_q\,, \qquad \psi^{1+}_{-2q} = \frac{i}{4}\Omega^{+2i}_{-}\gamma^i_{q\dot q}\chi^1_{\dot q}\,, \qquad \nabla_{-}\chi^1_q = \frac{1}{2}\Omega^{+2i}_{+}\gamma^i_{q\dot q}\chi^1_{\dot q}\,; \tag{4.14}$$

$$\psi^{2-}_{-2q} = -\frac{i}{2}\nabla_{-}\chi^2_q\,, \qquad \psi^{2-}_{+2q} = -\frac{i}{4}\Omega^{-2i}_{+}\gamma^i_{q\dot q}\chi^2_{\dot q}\,, \qquad \nabla_{+}\chi^2_q = -\frac{1}{2}\Omega^{-2i}_{-}\gamma^i_{q\dot q}\chi^2_{\dot q}\,; \tag{4.15}$$

$$\psi^{1-}_{-2\dot q} = -\frac{i}{2}\nabla_{-}\chi^1_{\dot q}\,, \qquad \psi^{1-}_{+2\dot q} = \frac{i}{4}\Omega^{-2i}_{+}\tilde\gamma^i_{\dot q q}\chi^1_q\,, \qquad \nabla_{+}\chi^1_{\dot q} = \frac{1}{2}\Omega^{-2i}_{-}\tilde\gamma^i_{\dot q q}\chi^1_q\,; \tag{4.16}$$

$$\psi^{2+}_{+2\dot q} = -\frac{i}{2}\nabla_{+}\chi^2_{\dot q}\,, \qquad \psi^{2+}_{-2\dot q} = -\frac{i}{4}\Omega^{+2i}_{-}\tilde\gamma^i_{\dot q q}\chi^2_q\,, \qquad \nabla_{-}\chi^2_{\dot q} = -\frac{1}{2}\Omega^{+2i}_{+}\tilde\gamma^i_{\dot q q}\chi^2_q\,. \tag{4.17}$$

Spinor-vector and vector-vector component equations of (4.7)–(4.10) are the consequences of the adduced ones. Then the scalar superfield S that enters expressions for the induced supertorsion can be represented as

$$S = \frac{i}{8}\left(\Omega^{+2i}_{+}\Omega^{-2i}_{-} + \Omega^{+2i}_{-}\Omega^{-2i}_{+}\right). \tag{4.18}$$

Similarly to the $D=3$ $N=2$ target superspace case [28] considered superembedding is off-shell. This can be easily seen from eqs. (4.16) and (4.17)

$$\psi^{1-}_{-2\dot q} = -\frac{i}{2}\nabla_{-}\chi^1_{\dot q}\,, \qquad \psi^{2+}_{+2\dot q} = -\frac{i}{2}\nabla_{+}\chi^2_{\dot q} \tag{4.19}$$

that have non-zero r.h.s. compared to the type-IIA superstring fermionic equations of motion (3.31).

It is possible to consider the case when the auxiliary spinor superfields $\lambda_q^{1,2}(z^M)$ are identified with the superfields $\chi_q^{1,2}(z^M)$ originating from the decompositions (2.23) and (2.24)

$$\lambda_q^1 = \chi_q^1\,, \qquad \lambda_q^2 = \chi_q^2\,. \tag{4.20}$$

This scheme can be regarded as the "minimal" one in the sence that it involves only the superfields originating from the decompositions of the target space supervielbein over the superzweinbein basis. From (2.29) and (2.30) we deduce that $\lambda_q^{1,2}$ possess the unit norm. Let us examine the situation when in addition the worldsheet supergravity constraints (4.11)–(4.12) are imposed. Then from (4.13) it follows that

$$\chi_{\dot q}^1 = \chi_{\dot q}^2 = 0 \tag{4.21}$$

and consequently

$$\psi^{1+}_{-2q} = \psi^{2-}_{+2q} = 0\,, \qquad \psi^{1-}_{-2\dot q} = \psi^{2+}_{+2\dot q} = 0\,, \tag{4.22}$$

$$\Omega_+^{+2i} = \Omega_-^{-2i} = 0\,, \qquad \Omega_{+2}^{+2i} = \Omega_{-2}^{-2i}\,. \tag{4.23}$$

Further utilization of Codazzi equations (2.33) allows to deduce that $\Omega_{+2}^{+2i} = \Omega_{-2}^{-2i} = 0$. Thus we observe that the superstring equations of motion are satisfied, so the "minimal" embedding is on-shell.

In the end of the section let us show how the relation between $\kappa-$symmetry gauge fixed physical variables of the NSR string and the type-IIA superstring arises. Substituting the representation for the NSR string transverse Grassmann variables $\varphi_\pm^i$

$$\varphi_\pm^i = i\left(\nabla_\pm x^i - \frac{1}{2}\Omega_\pm^{+2i}x^{-2} - \frac{1}{2}\Omega_\pm^{-2i}x^{+2}\right) \tag{4.24}$$

into expressions for $\pm$ components of the embedding equation $\Pi^i = 0$ (2.18) and using (4.13) and (4.21) we derive

$$\varphi_+^i = \lambda_q^1\gamma^i_{q\dot q}\theta^{1-}_{\dot q}\,, \qquad \varphi_-^i = -\lambda_q^2\gamma^i_{q\dot q}\theta^{2+}_{\dot q}\,, \tag{4.25}$$

which leading components are just the relations between the physical variables of the NSR string and the type-IIA superstring, as was argued in [54].

5. Embedding of the $n = (8,8)$ superworldsheet into $D = 10$ $N = 2B$ target superspace

In the case under consideration basic equations governing the superembedding are (2.18) and (2.20).

Induced supertorsion 2-form is given by the following expressions

$$T^{+2} = -2ie_q^+e_q^+ - 2i\pi_q^{2+}\pi_q^{2+}\,, \qquad T^{-2} = -2i\pi_{\dot q}^{1-}\pi_{\dot q}^{1-} - 2ie_{\dot q}^-e_{\dot q}^-\,, \tag{5.1}$$

$$T_q^+ = -\frac{1}{2}\gamma^i_{q\dot p}\Omega^{+2i}\pi_{\dot p}^{1-}\,, \qquad T_{\dot q}^- = -\frac{1}{2}\tilde\gamma^i_{\dot q p}\Omega^{-2i}\pi_p^{2+}\,. \tag{5.2}$$

There also arises the 2-form equation

$$-\frac{1}{2}\Omega^{+2i}e^{-2} - \frac{1}{2}\Omega^{-2i}e^{+2} - 2ie_q^+\gamma^i_{q\dot q}\pi^{1-}_{\dot q} - 2i\pi_q^{2+}\gamma^i_{q\dot q}e^-_{\dot q} = 0\,. \tag{5.3}$$

Target superspace fermionic 1-forms $\pi^{1-}_{\dot q}$ and π^{2+}_{q} that have not been identified with the superzweinbein can be expanded over the worldsheet basis as

$$\begin{aligned}\pi^{1-}_{\dot q} &= e^{+2}\psi^{1-}_{+2\dot q} + e^{-2}\psi^{1-}_{-2\dot q} + e^{+}_{p}h^{1-}_{+p\dot q} + e^{-}_{\dot p}\chi^{1}_{\dot p\dot q}\,,\\ \pi^{2+}_{q} &= e^{+2}\psi^{2+}_{+2q} + e^{-2}\psi^{2+}_{-2q} + e^{+}_{p}\chi^{2}_{pq} + e^{-}_{\dot p}h^{+2}_{-\dot pq}\,.\end{aligned} \tag{5.4}$$

The integrability conditions for these expansions read

$$\begin{aligned}&(e^{+2}\nabla + T^{+2})\psi^{1-}_{+2\dot q} + (e^{-2}\nabla + T^{-2})\psi^{1-}_{-2\dot q} + (e^{+}_{p}\nabla + T^{+}_{p})h^{1-}_{+p\dot q} + (e^{-}_{\dot p}\nabla + T^{-}_{\dot p})\chi^{1}_{\dot p\dot q} =\\ &\quad = -\frac{1}{2}\tilde\gamma^{i}_{\dot q q}\Omega^{-2i}e^{+}_{q}\,,\end{aligned} \tag{5.5}$$

$$\begin{aligned}&(e^{+2}\nabla + T^{+2})\psi^{2+}_{+2q} + (e^{-2}\nabla + T^{-2})\psi^{2+}_{-2q} + (e^{+}_{p}\nabla + T^{+}_{p})\chi^{2}_{pq} + (e^{-}_{\dot p}\nabla + T^{-}_{\dot p})h^{2+}_{-\dot pq} =\\ &\quad = -\frac{1}{2}\gamma^{i}_{q\dot q}\Omega^{+2i}e^{-}_{\dot q}\,.\end{aligned} \tag{5.6}$$

The analysis of the derived integrability conditions proceeds in the same way as in section 3. The spinor-spinor components of eq. (5.3) yield that

$$h^{1-}_{+p\dot q} = h^{2+}_{-\dot pq} = 0\,, \qquad \chi^{1}_{\dot q\dot p} = -\chi\delta_{\dot q\dot p}\,, \qquad \chi^{2}_{qp} = \chi\delta_{qp}\,. \tag{5.7}$$

The non-zero superfield $\chi(z^M)$ as will be shown below is a constant and is related to the parameter of the global SO(2) rotations of the target superspace Grassmann coordinates $\Theta^{1,2\underline{\mu}}$. As is well known, such rotations are the symmetry of the embedding equations rather than the type-IIB GS superstring action. Recently it was established that χ is related to the on-shell value of the field strength of $2d$ gauge field living on the D1-brane, so that superembedding under consideration describes on equal footing both the type-IIB superstring and the D1-brane [67].

From the integrability conditions (5.5) and (5.6) one infers that

$$\begin{aligned}d\chi &= 0\,, \qquad \psi^{1-}_{-2\dot q} = \psi^{2+}_{+2q} = \nabla_{-\dot p}\psi^{1-}_{+2\dot q} = \nabla_{+p}\psi^{2+}_{-2q} = 0\,,\\ \psi^{1-}_{+2\dot q} &= \frac{i}{32}\tilde\gamma^{i}_{\dot qp}\Omega^{-2i}_{+p}\,, \qquad \psi^{2+}_{-2q} = \frac{i}{32}\gamma^{i}_{q\dot p}\Omega^{+2i}_{-\dot p}\,,\end{aligned} \tag{5.8}$$

$$\Omega^{+2i}_{+p} = \Omega^{-2i}_{-\dot p} = 0\,, \qquad \Omega^{+2i}_{+2} = \Omega^{-2i}_{-2} = \frac{4i\chi}{(1+\chi^2)}\psi^{1-}_{+2\dot q}\tilde\gamma^{i}_{\dot qp}\psi^{2+}_{-2p}\,, \tag{5.9}$$

$$\Omega^{-2i}_{+2} = -\frac{1}{4(1+\chi^2)}\nabla_{+q}\gamma^{i}_{q\dot p}\psi^{1-}_{+2\dot p}\,, \qquad \Omega^{+2i}_{-2} = -\frac{1}{4(1+\chi^2)}\nabla_{-\dot p}\tilde\gamma^{i}_{\dot pq}\psi^{2+}_{-2q}\,. \tag{5.10}$$

Note, that in the presence of parameter χ the superworldsheet mean curvature is non zero.

Non-zero supertorsion components look like

$$\begin{aligned}T^{+2}_{-2+q} &= -4i\chi\psi^{2+}_{-2q}\,, \qquad T^{+2}_{+q+p} = 4i\delta_{qp}(1+\chi^2)\,;\\ T^{-2}_{+2-\dot q} &= 4i\chi\psi^{1-}_{+2\dot q}\,, \qquad T^{-2}_{-\dot q-\dot p} = 4i\delta_{\dot q\dot p}(1+\chi^2)\,;\\ T^{+q}_{+2-2} &= -\frac{1}{2}\gamma^{i}_{q\dot p}\Omega^{+2i}_{-2}\psi^{1-}_{+2\dot p}\,,\\ T^{+q}_{-\dot p+2} &= \frac{1}{2}\gamma^{i}_{q\dot q}\Omega^{+2i}_{-\dot p}\psi^{1-}_{+2\dot q} + \frac{\chi}{2}\gamma^{i}_{q\dot p}\Omega^{+2i}_{+2}\,, \qquad T^{+q}_{-\dot p-2} = \frac{\chi}{2}\gamma^{i}_{q\dot p}\Omega^{+2i}_{-2}\,,\end{aligned}$$

$$T^{+q}{}_{-\dot{q}-\dot{p}} = -\frac{\chi}{2}\left(\gamma^i_{q\dot{q}}\Omega^{+2i}_{-\dot{p}} + \dot{q} \leftrightarrow \dot{p}\right);$$
$$T^{-\dot{q}}_{+2-2} = \frac{1}{2}\tilde{\gamma}^i_{\dot{q}p}\Omega^{-2i}_{+2}\psi^{2+}_{-2p},$$
$$T^{-\dot{q}}_{+p+2} = -\frac{\chi}{2}\tilde{\gamma}^i_{\dot{q}p}\Omega^{-2i}_{+2}, \qquad T^{-\dot{q}}_{+p-2} = -\frac{\chi}{2}\tilde{\gamma}^i_{\dot{q}p}\Omega^{-2i}_{-2} + \frac{1}{2}\tilde{\gamma}^i_{\dot{q}q}\Omega^{-2i}_{+p}\psi^{2+}_{-2q},$$
$$T^{-\dot{q}}_{+q+p} = \frac{\chi}{2}\left(\tilde{\gamma}^i_{\dot{q}q}\Omega^{-2i}_{+p} + q \leftrightarrow p\right). \tag{5.11}$$

Similarly to the type-IIA case there survive only two chiral superfields $\psi^{1-}_{+2\dot{q}}(z^M)$ and $\psi^{2+}_{-2q}(z^M)$. The peculiarity of the type-IIB superembedding is the presence of the constant parameter χ.

Analogously to the type-IIA case we can obtain the supersymmetric generalization of the type-IIB superstring equations of motion, and show that for the superstring (i.e. when $\chi = 0$) the embedding is on-shell.

6. Embedding of the $n = (1,1)$ superworldsheet into $D = 10$ $N = 2B$ target superspace

This superembedding is described by equations are (2.18) and (2.22).

The derivation of the integrability conditions for the basic superembedding equations (2.18), (2.22), (2.23), (2.25), (2.26) and (2.28) is carried out among the lines of section 3. From eqs. (2.18) and (2.22) one deduces the expressions for the induced supertorsion

$$T^{+2} = -2i\pi^{1+}_q\pi^{1+}_q - 2i\pi^{2+}_q\pi^{2+}_q, \qquad T^{-2} = -2i\pi^{1-}_{\dot{q}}\pi^{1-}_{\dot{q}} - 2i\pi^{2-}_{\dot{q}}\pi^{2-}_{\dot{q}}, \tag{6.1}$$
$$T^{+} = \pi^{1+}_q\nabla\lambda^1_q - \frac{1}{2}\lambda^1_q\gamma^i_{q\dot{q}}\Omega^{+2i}\pi^{1-}_{\dot{q}}, \qquad T^{-} = \pi^{2-}_{\dot{q}}\nabla\lambda^2_{\dot{q}} - \frac{1}{2}\lambda^2_{\dot{q}}\tilde{\gamma}^i_{\dot{q}q}\Omega^{-2i}\pi^{2+}_q \tag{6.2}$$

and the equation

$$-\frac{1}{2}\Omega^{+2i}e^{-2} - \frac{1}{2}\Omega^{-2i}e^{+2} - 2i\pi^{1+}_q\gamma^i_{q\dot{q}}\pi^{1-}_{\dot{q}} - 2i\pi^{2+}_q\gamma^i_{q\dot{q}}\pi^{2-}_{\dot{q}} = 0. \tag{6.3}$$

These equations should be supplemented by the integrability conditions for (2.23), (2.25), (2.26) and (2.28) that coincide with eqs. (4.7) and (4.9) and those obtained by the substitution $(1 \leftrightarrow 2)$.

As the result of the imposition of $2d$ supergravity constraints (4.11) and (4.12) into the l.h.s. of (6.1) and (6.2) one infers the following restrictions on the superfield coefficients of (2.23), (2.25), (2.26) and (2.28)

$$h^{1,2+}_{-q} = h^{1,2-}_{+\dot{q}} = 0, \qquad \chi^1_q\chi^1_q + \chi^2_q\chi^2_q = 1, \qquad \chi^1_{\dot{q}}\chi^1_{\dot{q}} + \chi^2_{\dot{q}}\chi^2_{\dot{q}} = 1. \tag{6.4}$$

Analogously to the type-IIA target superspace case considered $n = (1,1)$ superworldsheet embedding is off-shell.

One is able to consider the "minimal" embedding scheme (cf. section 4), where λ^1_q and $\lambda^2_{\dot{q}}$ are identified with χ^1_q and $\chi^2_{\dot{q}}$ respectively

$$\lambda^1_q = \chi^1_q, \qquad \lambda^2_{\dot{q}} = \chi^2_{\dot{q}}, \tag{6.5}$$

so that

$$\lambda_q^1 \lambda_q^1 = \lambda_{\dot q}^2 \lambda_{\dot q}^2 = 1\,. \tag{6.6}$$

When additionally the worldsheet supergravity constraints are imposed it follows from (6.4) that

$$\chi_q^2 = \chi_{\dot q}^1 = 0 \tag{6.7}$$

and consequently

$$\psi^{1+}_{-2q} = \psi^{2+}_{+2q} = 0\,, \qquad \psi^{1-}_{-2\dot q} = \psi^{2-}_{+2\dot q} = 0\,. \tag{6.8}$$

It is possible to verify that the "minimal" embedding is on-shell in the same way as was done in section 4.

Applying the same arguments as in section 3 we find the relation between the $\kappa-$symmetry gauge fixed physical variables of the NSR string and the type-IIB superstring from equation $\Pi^i = 0$ (2.18)

$$\varphi^i_+ = \lambda^1_q \gamma^i_{q\dot q} \theta^{1-}_{\dot q}\,, \qquad \varphi^i_- = \lambda^2_{\dot q} \tilde\gamma^i_{\dot q q} \theta^{2+}_q\,, \tag{6.9}$$

which leading components are precisely the relations obtained in [54] in the framework of the twistor-like Lorentz harmonic description of type-II superstrings.

7. Conclusion

We have considered possible mechanisms of generalization of the superembedding approach technique to the case when the number of the worldvolume Grassmann directions is less than $n_{\rm ts}/2$, $n_{\rm ts}$ being the number of the target space supersymmetries, on example of $n = (1,1)$ string superworldsheet embedded into $D = 10$ type-II superspace. We formulated basic equations governing the superembedding and derived their integrability conditions. The consequences of the $2d$ $n = (1,1)$ supergravity constraints imposition on the worldsheet induced supertorsion were analysed. Unlike the on-shell $n = (8,8)$ superworldsheet embedding, the $n = (1,1)$ superworldsheet embedding is off-shell. There was also proposed the "minimal" superembedding scheme for which auxiliary spinor superfields are taken from the decomposition coefficients of the target space superforms over the superworldsheet basis. The imposition of the worldsheet supergravity constraints puts the "minimal" embedding on the mass shell. We have also shown how the covariant relation between the physical variables of the NSR string and type-II superstrings originates in the framework of the "minimal" superembedding.

Construction of the corresponding worldsheet superfiled action that should generalize $D = 3$ $N = 2$ one of ref. [28], as well as, the extension of the proposed mechanisms of superembedding for diverse numbers of worldsheet supersymmetries and other branes can be the subject for future research. Another domain for exploration is a classification of such embeddings with $n_{\rm wv} < n_{\rm ts}/2$ according to a fraction of target space supersymmetries they preserve.

JHEP07(2002)008

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References

[1] J. Hughes, J. Liu and J. Polchinski, *Supermembranes*, *Phys. Lett.* **B 180** (1986) 370;
J. Hughes and J. Polchinski, *Partially broken global supersymmetry and the superstring*, *Nucl. Phys.* **B 278** (1986) 147.

[2] M.J. Duff, R.R. Khuri and J.X. Lu, *String solitons*, *Phys. Rept.* **259** (1995) 213 [hep-th/9412184];
M.J. Duff, *Supermembranes*, hep-th/9611203.

[3] D.V. Volkov and A. Zheltukhin, *Extension of the Penrose representation and its use to describe supersymmetric models*, *JETP Lett.* **48** (1988) 63; *On the equivalence of the lagrangians of massless Dirac and supersymmetrical particles*, *Lett. Math. Phys.* **17** (1989) 141; *Lagrangians for massless particles and strings with local and global supersymmetry*, *Nucl. Phys.* **B 335** (1990) 723.

[4] D. Sorokin, V. Tkach and D.V. Volkov, *Superparticles, twistors and Siegel symmetry*, *Mod. Phys. Lett.* **A 4** (1989) 901.

[5] D.P. Sorokin, V.I. Tkach, D.V. Volkov and A.A. Zheltukhin, *From the superparticle Siegel symmetry to the spinning particle proper time supersymmetry*, *Phys. Lett.* **B 216** (1989) 302.

[6] S.J. Gates and H. Nishino, *$D = 2$ superfield supergravity, local (supersymmetry)^2 and nonlinear sigma models*, *Class. and Quant. Grav.* **3** (1986) 391;
J. Kowalski-Glikman, *Doubly graded sigma model with torsion*, *Phys. Lett.* **B 180** (1986) 358;
R. Brooks, F. Muhammed and S.J. Gates, *Matter coupled to $D = 2$ simple unidexterous supergravity, local (supersymmetry)^2 and strings*, *Class. and Quant. Grav.* **3** (1986) 745;
R. Brooks, *Superstrings from the ((1,0) superspace)^2 nonlinear sigma model*, *Phys. Lett.* **B 186** (1987) 313;
J. Kowalski-Glikman, J.W. van Holten, S. Aoyama and J. Lukierski, *The spinning superparticle*, *Phys. Lett.* **B 201** (1988) 487;
A. Kavalov and R.L. Mkrtychyan, *Spinning superparticles*, preprint YerPhI 1068(31)-88, Yerevan, 1988;
J.M.L. Fisch, *The $N = 1$ spinning superstring: a model of the super superstring*, *Phys. Lett.* **B 219** (1989) 71.

[7] A. Galajinsky and O. Lechtenfeld, *Towards a stringy extension of self-dual super Yang-Mills*, *Phys. Lett.* **B 460** (1999) 288 [hep-th/9903149].

[8] D.P. Sorokin, *Double supersymmetric particle theories*, *Fortschr. Phys.* **38** (1990) 923.

[9] P.S. Howe and P.K. Townsend, *The massless superparticle as Chern-Simons mechanics*, *Phys. Lett.* **B 259** (1991) 285.

[10] J.P. Gauntlett, *A kappa symmetry calculus for superparticles*, *Phys. Lett.* **B 272** (1991) 25 [hep-th/9109039].

[11] F. Delduc and E. Sokatchev, *Superparticle with extended worldline supersymmetry*, *Class. and Quant. Grav.* **9** (1992) 361.

[12] A. Galperin and E. Sokatchev, *A twistor like $D = 10$ superparticle action with manifest $N = 8$ worldline supersymmetry*, *Phys. Rev.* **D 46** (1992) 714 [hep-th/9203051].

[13] A. Pashnev and D. Sorokin, *Note on superfield formulations of $D = 2$, $D = 3$, $D = 4$, $D = 6$ and $D = 10$ superparticles*, *Class. and Quant. Grav.* **10** (1993) 625.

[14] I.A. Bandos, A.Yu. Nurmagambetov, D.P. Sorokin and D.V. Volkov, *On another version of the twistor-like approach to superparticles*, *JETP Lett.* **60** (1994) 621 [hep-ph/9409439].

[15] I.A. Bandos, A.Yu. Nurmagambetov, D.P. Sorokin and D.V. Volkov, *Twistor-like superparticles revisited*, *Class. and Quant. Grav.* **12** (1995) 1881 [hep-th/9502143].

[16] I. Bandos, A. Maznytsia and D. Sorokin, *Worldline superfield actions for $N = 2$ superparticles*, *Int. J. Mod. Phys.* **A 14** (1999) 1975 [hep-th/9711007].

[17] I.A. Bandos, D.P. Sorokin, M. Tonin and D.V. Volkov, *Doubly supersymmetric null strings and string tension generation*, *Phys. Lett.* **B 319** (1993) 445 [hep-th/9307039].

[18] N. Berkovits, *A covariant action for the heterotic superstring with manifest space-time supersymmetry and world sheet superconformal invariance*, *Phys. Lett.* **B 232** (1989) 184; *A geometrical interpretation for the symmetries of the free Green-Schwarz heterotic superstring*, *Phys. Lett.* **B 241** (1990) 497.

[19] M. Tonin, *World sheet supersymmetric formulations of Green-Schwarz superstrings*, *Phys. Lett.* **B 266** (1991) 312; *Kappa symmetry as world sheet supersymmetry in $D = 10$ heterotic superstring*, *Int. J. Mod. Phys.* **A 7** (1992) 6013.

[20] E.A. Ivanov and A.A. Kapustnikov, *Towards a tensor calculus for kappa supersymmetry*, *Phys. Lett.* **B 267** (1991) 175.

[21] N. Berkovits, *The heterotic Green-Schwarz superstring on an $N = (2, 0)$ superworldsheet*, *Nucl. Phys.* **B 379** (1992) 96 [hep-th/9201004].

[22] N. Berkovits, *A new sigma model action for the four-dimensional Green-Schwarz heterotic superstring*, *Phys. Lett.* **B 304** (1993) 249 [hep-th/9303025].

[23] F. Delduc, E. Ivanov and E. Sokatchev, *Twistor like superstrings with $D = 3$, $D = 4$, $D = 6$ target superspace and $N = (1, 0)$, $N = (2, 0)$, $N = (4, 0)$ world sheet supersymmetry*, *Nucl. Phys.* **B 384** (1992) 334 [hep-th/9204071].

[24] F. Delduc, A. Galperin, P.S. Howe and E. Sokatchev, *A twistor formulation of the heterotic $d = 10$ superstring with manifest (8,0) world sheet supersymmetry*, *Phys. Rev.* **D 47** (1993) 578 [hep-th/9207050].

[25] I.A. Bandos, M. Cederwall, D.P. Sorokin and D.V. Volkov, *Towards a complete twistorization of the heterotic string*, *Mod. Phys. Lett.* **A 9** (1994) 2987 [hep-th/9403181].

[26] P.S. Howe, *A note on chiral fermions and heterotic strings*, *Phys. Lett.* **B 332** (1994) 61 [hep-th/9403177].

[27] I. Bandos and T. Bandos, *Lorentz harmonics and superfield action: $D = 10$, $N = 1$ superstring*, *Class. and Quant. Grav.* **18** (2001) 1907 [hep-th/0010044].

[28] A. Galperin and E. Sokatchev, *A twistor formulation of the nonheterotic superstring with manifest world sheet supersymmetry*, *Phys. Rev.* **D 48** (1993) 4810 [hep-th/9304046].

JHEP07(2002)008

[29] V. Chikalov and A. Pashnev, *Twistor like type-II superstring and bosonic string*, *Mod. Phys. Lett.* **A 8** (1993) 285 [hep-th/9209115]; *Twistor like type-II superstring with the heterotic (2,0) and (4,0) world sheet supersymmetry*, *Phys. Rev.* **D 50** (1994) 7450.

[30] P.S. Howe, O. Raetzel and E. Sezgin, *On brane actions and superembeddings*, *J. High Energy Phys.* **08** (1998) 011 [hep-th/9804051].

[31] P. Pasti, D.P. Sorokin and M. Tonin, *Superembeddings, partial supersymmetry breaking and superbranes*, *Nucl. Phys.* **B 591** (2000) 109 [hep-th/0007048]; *Geometrical aspects of superbrane dynamics*, *Fortschr. Phys.* **49** (2001) 649 [hep-th/0011020].

[32] I. Bandos, P. Pasti, A. Pokotilov, D. Sorokin, M. Tonin, *The space filling Dirichlet 3-brane in $N = 2$, $D = 4$ superspace*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 18 [hep-th/0103152].

[33] J.M. Drummond and P.S. Howe, *Codimension zero superembeddings*, *Class. and Quant. Grav.* **18** (2001) 4477 [hep-th/0103191].

[34] I.A. Bandos, D.P. Sorokin, M. Tonin, P. Pasti and D.V. Volkov, *Superstrings and supermembranes in the doubly supersymmetric geometrical approach*, *Nucl. Phys.* **B 446** (1995) 79 [hep-th/9501113].

[35] P.S. Howe and E. Sezgin, $D = 11$, $p = 5$, *Phys. Lett.* **B 394** (1997) 62 [hep-th/9611008];
P. Pasti, D.P. Sorokin and M. Tonin, *Covariant action for a $D = 11$ five-brane with the chiral field*, *Phys. Lett.* **B 398** (1997) 41 [hep-th/9701037];
P.S. Howe, E. Sezgin and P.C. West, *Covariant field equations of the M-theory five-brane*, *Phys. Lett.* **B 399** (1997) 49 [hep-th/9702008];
I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, *Covariant action for the super-five-brane of M-theory*, *Phys. Rev. Lett.* **78** (1997) 4332 [hep-th/9701149];
M. Aganagic, J. Park, C. Popescu and J.H. Schwarz, *World-volume action of the M-theory five-brane*, *Nucl. Phys.* **B 496** (1997) 191 [hep-th/9701166];
I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, *On the equivalence of different formulations of the M-theory five-brane*, *Phys. Lett.* **B 408** (1997) 135 [hep-th/9703127].

[36] D.P. Sorokin, *Superbranes and superembeddings*, *Phys. Rept.* **329** (2000) 1 [hep-th/9906142]; *Introduction to the superembedding description of superbranes*, hep-th/0105102.

[37] P.S. Howe, E. Sezgin and P.C. West, *Aspects of superembeddings*, hep-th/9705093.

[38] L. Eisenhart, *Riemannian geometry*, Princeton University Press, NJ 1964.

[39] F. Lund and T. Regge, *Unified approach to strings and vortices with soliton solutions*, *Phys. Rev.* **D 14** (1976) 1524;
R. Omnes, *A new geometric approach to the relativistic string*, *Nucl. Phys.* **B 149** (1979) 269.

[40] B.M. Barbashov and V.V. Nesterenko, *Introduction to relativistic string theory*, World Scientific, Singapore 1990.

[41] A.A. Zheltukhin, *Connection between a relativistic string and two-dimensional field models*, *Sov. J. Nucl. Phys.* **33** (1981) 927; *Classical relativistic string as a two-dimensional* $\mathrm{SO}(1,1) \times \mathrm{SO}(2)$ *gauge model*, *Theor. Math. Phys.* **52** (1982) 666; *Gauge description and nonlinear string equations in d-dimensional space-time*, *Theor. Math. Phys.* **56** (1984) 785; *Classical relativistic string as an exactly solvable sector of the* $\mathrm{SO}(1,1) \times \mathrm{SO}(2)$ *gauge model*, *Phys. Lett.* **B 116** (1982) 147.

[42] E. Ivanov, *Superbranes and super Born-Infeld theories as nonlinear realizations*, *Theor. Math. Phys.* **129** (2001) 1543 [hep-th/0105210].

[43] P.S. Howe, A. Kaya, E. Sezgin and P. Sundell, *Codimension one branes*, *Nucl. Phys.* **B 587** (2000) 481 [hep-th/0001169].

[44] I.A. Bandos, D.P. Sorokin and D. Volkov, *On the generalized action principle for superstrings and supermembranes*, *Phys. Lett.* **B 352** (1995) 269 [hep-th/9502141].

[45] I.A. Bandos and A.Yu. Nurmagambetov, *Generalized action principle and extrinsic geometry for $N = 1$ superparticle*, *Class. and Quant. Grav.* **14** (1997) 1597 [hep-th/9610098].

[46] I.A. Bandos, *Doubly supersymmetric geometric approach for heterotic string from generalized action principle to exactly solvable nonlinear equations*, hep-th/9510213.

[47] A.Y. Nurmagambetov, *A generalized action principle for $D = 4$ doubly supersymmetric membrane*, hep-th/9512102.

[48] I.A. Bandos, D.P. Sorokin and M. Tonin, *Generalized action principle and superfield equations of motion for $d = 10$ Dp-branes*, *Nucl. Phys.* **B 497** (1997) 275 [hep-th/9701127].

[49] J.P. Gauntlett, G.W. Gibbons, C.M. Hull and P.K. Townsend, *BPS states of $D = 4$ $N = 1$ supersymmetry*, *Commun. Math. Phys.* **216** (2001) 431 [hep-th/0001024];
I. Bandos and J. Lukierski, *Tensorial central charges and new superparticle models with fundamental spinor coordinates*, *Mod. Phys. Lett.* **A 14** (1999) 1257 [hep-th/9811022];
I. Bandos, J. Lukierski and D.P. Sorokin, *Superparticle models with tensorial central charges*, *Phys. Rev.* **D 61** (2000) 045002 [hep-th/9904109]; *The* $\mathrm{OSp}(1|4)$ *superparticle and exotic BPS states*, hep-th/9912264;
S. Fedoruk and V.G. Zima, *Massive superparticle with tensorial central charges*, *Mod. Phys. Lett.* **A 15** (2000) 2281 [hep-th/0009166].

[50] A.A. Zheltukhin and U. Lindström, *Strings in a space with tensor central charge coordinates*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 126 [hep-th/0103101]; *Hamiltonian of tensionless strings with tensor central charge coordinates*, *J. High Energy Phys.* **01** (2002) 034 [hep-th/0112206];
A.A. Zheltukhin and D.V. Uvarov, *An inverse Penrose limit and supersymmetry enhancement in the presence of tensor central charges*, hep-th/0206214.

[51] S. Aoyama, P. Pasti and M. Tonin, *The GS and NRS heterotic strings from twistor string models*, *Phys. Lett.* **B 283** (1992) 213.

[52] N. Berkovits, *The ten-dimensional Green-Schwarz superstring is a twisted Neveu-Schwarz-Ramond string*, *Nucl. Phys.* **B 420** (1994) 332 [hep-th/9308129]; *Covariant quantization of the Green-Schwarz superstring in a Calabi-Yau background*, *Nucl. Phys.* **B 431** (1994) 258 [hep-th/9404162]; *Extra dimensions in superstring theory*, *Nucl. Phys.* **B 507** (1997) 731 [hep-th/9704109]; *Quantization of the superstring with manifest* $\mathrm{U}(5)$ *super-Poincaré invariance*, *Phys. Lett.* **B 457** (1999) 94 [hep-th/9902099]; *Super-Poincaré covariant quantization of the superstring*, *J. High Energy Phys.* **04** (2000) 018 [hep-th/0001035]; *Covariant quantization of the superstring*, *Int. J. Mod. Phys.* **A 16** (2001) 801 [hep-th/0008145]; *Relating the rns and pure spinor formalisms for the superstring*, *J. High Energy Phys.* **08** (2001) 026 [hep-th/0104247];
N. Berkovits and C. Vafa, *$N = 4$ topological strings*, *Nucl. Phys.* **B 433** (1995) 123 [hep-th/9407190].

JHEP07(2002)008

[53] D. Polyakov, *Picture changing operators and space-time supersymmetry*, *Nucl. Phys.* **B 449** (1995) 159 [hep-th/9502124]; *Kappa symmetry and Bogomol'ny bound*, *Nucl. Phys.* **B 485** (1997) 128 [hep-th/9609092].

[54] D.V. Uvarov, *On covariant kappa-symmetry fixing and the relation between the NSR string and the type-II GS superstring*, *Phys. Lett.* **B 493** (2000) 421 [hep-th/0006185]; *Covariant kappa symmetry gauge fixing and the classical relation between physical variables of the NSR string and the type-II GS superstring*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 120 [hep-th/0104235].

[55] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, *Unconstrained $N = 2$ matter, Yang-Mills and supergravity theories in harmonic superspace*, *Class. and Quant. Grav.* **1** (1984) 469; *Unconstrained off-shell $N = 3$ supersymmetric Yang-Mills theory*, *Class. and Quant. Grav.* **2** (1985) 155.

[56] E. Sokatchev, *Light cone harmonic superspace and its applications*, *Phys. Lett.* **B 169** (1986) 209; *Harmonic superparticle*, *Class. and Quant. Grav.* **4** (1987) 237.

[57] E. Nissimov, S. Pacheva and S. Solomon, *Covariant first and second quantization of the $N = 2$ $D = 10$ Brink-Schwarz superparticle*, *Nucl. Phys.* **B 296** (1988) 462; *Covariant canonical quantization of the Green-Schwarz superstring*, *Nucl. Phys.* **B 297** (1988) 349; *Covariant unconstrained superfield action for the linearized $D = 10$ super Yang-Mills theory*, *Nucl. Phys.* **B 299** (1988) 183; *Off-shell superspace $D = 10$ super Yang-Mills from covariantly quantized Green-Schwarz superstring*, *Nucl. Phys.* **B 317** (1989) 344; *The relation between operator and path integral covariant quantizations of the green-schwarz superstring*, *Phys. Lett.* **B 228** (1989) 181;
E.R. Nissimov and S.J. Pacheva, *Cancellation of anomalies in the super-Poincaré covariant quantization of the Green-Schwarz superstring*, *Phys. Lett.* **B 221** (1989) 307.

[58] R. Kallosh and M. Rakhmanov, *Covariant quantization of the Green-Schwarz superstring*, *Phys. Lett.* **B 209** (1988) 233; *Consistency of covariant quantization of gs string*, *Phys. Lett.* **B 214** (1988) 549.

[59] P.B. Wiegmann, *Extrinsic geometry of superstrings*, *Nucl. Phys.* **B 323** (1989) 330.

[60] I.A. Bandos, *A superparticle in Lorentz-harmonic superspace*, *Sov. J. Nucl. Phys.* **51** (1990) 906; *Multivalued action functionals, Lorentz harmonics, and spin*, *JETP Lett.* **52** (1990) 205.

[61] I.A. Bandos and A.A. Zheltukhin, *Twistors, harmonics, and zero super-p-branes*, *JETP Lett.* **51** (1990) 618; *Quantum theory of closed null supermembranes in four-dimensional space*, *JETP Lett.* **53** (1991) 5; *Null super p-brane: hamiltonian dynamics and quantization*, *Phys. Lett.* **B 261** (1991) 245; *Covariant quantization of null supermembranes in four-dimensional space-time*, *Theor. Math. Phys.* **88** (1991) 925; *Null super p-branes quantum theory in four-dimensional space-time*, *Fortschr. Phys.* **41** (1993) 619.

[62] I.A. Bandos and A.A. Zheltukhin, *Spinor cartan moving N hedron, Lorentz harmonic formulations of superstrings, and kappa symmetry*, *JETP Lett.* **54** (1991) 421; *Green-Schwarz superstrings in spinor moving frame formalism*, *Phys. Lett.* **B 288** (1992) 77; *$D = 10$ superstring: lagrangian and hamiltonian mechanics in twistor-like Lorentz harmonic formulation*, *Phys. Part. Nucl.* **25** (1994) 453.

[63] I.A. Bandos and A.A. Zheltukhin, *Generalization of Newman-Penrose dyads in connection with the action integral for supermembranes in an eleven-dimensional space*, *JETP Lett.* **55** (1992) 81; *Eleven-dimensional supermembrane in a spinor moving Repere formalism*, *Int. J.*

Mod. Phys. **A 8** (1993) 1081; *Lorentz harmonics and new formulations of superstrings in $D = 10$ and supermembranes in $D = 11$*, *Phys. Atom. Nucl.* **56** (1993) 113; *$N = 1$ superp-branes in twistor-like Lorentz harmonic formulation*, *Class. and Quant. Grav.* **12** (1995) 609 [hep-th/9405113].

[64] F. Delduc, A. Galperin and E. Sokatchev, *Lorentz harmonic (super)fields and (super)particles*, *Nucl. Phys.* **B 368** (1992) 143;
A.S. Galperin, P.S. Howe and K.S. Stelle, *The superparticle and the Lorentz group*, *Nucl. Phys.* **B 368** (1992) 248 [hep-th/9201020].

[65] V.P. Akulov and D.V. Volkov, *Possible universal neutrino interaction*, *JETP Lett.* **16** (1972) 438; *Is the neutrino a Goldstone particle?*, *Phys. Lett.* **B 46** (1973) 109;
V.P. Akulov, D.V. Volkov and V.A. Soroka, *Gauge fields on superspaces with different holonomy groups*, *JETP Lett.* **22** (1975) 187;
J. Wess and B. Zumino, *Superspace formulation of supergravity*, *Phys. Lett.* **B 66** (1977) 361; *Superfield lagrangian for supergravity*, *Phys. Lett.* **B 74** (1978) 51.

[66] P.S. Howe, *Super Weyl transformations in two-dimensions*, *J. Phys.* **A 12** (1979) 393;
M. Brown and S.J. Gates, *Superspace Bianchi identities and the supercovariant derivative*, *Ann. Phys. (NY)* **122** (1979) 443.

[67] I. Bandos, *Superembedding approach and S-duality: a unified description of superstring and super-D1-brane*, *Nucl. Phys.* **B 599** (2001) 197 [hep-th/0008249].